Volatility Estimation of Stock Prices using Garch Method

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Abstract

Economic decisions are modeled based on perceived distribution of the random variables in the future, assessment and measurement of the variance which has a significant impact on the future profit or losses of particular portfolio. The ability to accurately measure and predict the stock market volatility has a wide spread implications. Volatility plays a very significant role in many financial decisions. The main purpose of this study is to examine the nature and the characteristics of stock market volatility of Kenyan stock markets and its stylized facts using GARCH models. Symmetric volatility model namely GARCH model (GARCH (1, 1) was used to estimate volatility of stock returns in Kenyan stock markets and its stylized facts including volatility clustering, fat tails and mean . The share prices used were obtained from Nairobi Stock Exchange specifically the share prices of Barclays Bank of Kenya running from 1st Jan, 2008 to 10th October, 2010. The results indicate the evidence of time varying stock return volatility over the sampled period of time. In conclusion, it follows that in volatility estimation, negative returns shocks have higher volatility than positive returns shocks.

Key words: GARCH, Stylized facts, Volatility clustering

Introduction

Volatility forecasting in financial market is very significant particularly in investment, financial risk management and monetary policy making (Poon and Granger, 2003). Volatility is defined as a degree of fluctuation in asset prices and it is an important phenomenon for traders and medium term - investors. Because of the link between volatility and risk, volatility can form a basis for efficient price discovery. Equilibrium prices derived from change in volatility affects asset pricing models and derivatives valuation which actually depends on a reliable volatility forecast. In recent years, volatility as a tool for measuring financial risks has been used to inform change in volatility of stock returns. In Financial Time series volatility clustering and leptokurtosis (fat tails) is commonly observed. Leverage effects are also observed in financial returns which occurs when the change in stock prices are negatively correlated with the changes in volatility. This kind of phenomena has led to the use of varying variance models to estimate and predict volatility of stock prices.

Modeling of time-varying conditional variance with Autoregressive Conditional Heteroskedasticity (ARCH) process was proposed by Engle (1982) using lagged disturbance. The results obtained from this work indicate that in order to capture the dynamic behaviour of conditional variance, a high order ARCH model is required. This problem was solved by Bollesrslev, (1986) by developing a Generalized ARCH model (GARCH) basing on infinite ARCH specifications which reduces the number of estimated parameters from infinity to two. Both ARCH and GARCH model capture volatility clustering and lepkurtosis but they fail to model leverage effects because their distribution is symmetric. To address this problem non-linear extension of GARCH has been proposed such as the exponential GARCH (EGARCH) model by Nelson, (1991) and also Asymmetric Power ARCH (APARCH) modeled by Ding

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et al, (1993). Another problem encountered when using GARCH is that they do not always fully embrace the thick tails property of a high frequency financial time series data.

The main objective of any investor is to maximize the expected returns or to minimize the risk subject to some constrains. Estimating volatility of an asset's price is a focal point for assessing the investment risk. Volatility of an underlying asset must be understood before any pricing is undertaken. This is because volatility has a great influence on the macro-economy and therefore if volatility increases beyond a certain threshold it increases the probability of great loses by investors raising concerns about the stability of the market and the economy at large (Hongyu and Zhichao, 2006).

Hansen and Lunde (2005) argued that GARCH (1, 1) works well in estimating volatility of financial returns as compared to more complicated models including EGARCH, GJR-GARCH etc. In modeling volatility of Chinese stock market, Hung-Chung et al. (2009) showed that GARCH model with an underlying leptokurtic asymmetric distribution has better forecasting ability as compared to an underlying normal distribution. The use of GARCH (1, 1) model with a fat tail error distribution leads to an improvement in Volatility forecasting (Wilhemsson, 2006). Student's t distribution as a distribution assumption to a GARCH model outperforms the exponential distribution and a mixture of normal distribution (Chuanga et al., 2007).

The forecasting ability of symmetric and asymmetric GARCH models was compared by Balaban (2004), Specifically GARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1) volatility equations were used and established that that EGARCH model performs better on the out of sample data forecast, followed by GARCH and GJR-GARCH (1, 1) in that order.

Volatility and forecasting market risks from observations from Egypt (CMA and general index) and Israel (TASE-100 Index) was modeled using GARCH by Floros, (2008). It was found that Egyptian CMA Index was the most volatile series due to prices (and economy) uncertainty during the time period under consideration.

Volatility clustering, excess kurtosis and heavy tails of time series of KSE using ARCH and GARCH was studied Rafique and Kashif-ur-Rehman, (2011).

Preliminary Notes

The model describing the returns of an asset at time t can be defined as;

$$Y_t = \mu + \sigma_t \varepsilon_t \ , \quad t \in \Re \tag{1}$$

where , $\{\sigma_t\}$ is non-negative stochastic process such that for a fixed t, ε_t and σ_t are independent and $\{\varepsilon_t\}$ is a sequence of identically distributed and symmetric random variables. Volatility process is identified by $\{\sigma_t\}$.

The time series $\{Y_t\}$ and the volatility process $\{\sigma_t\}$ are assumed to be strictly stationary and the mean μ is assumed to be zero.

The movement of the price changes can be modelled only by the sign of ε_t which is independent of the order of magnitude of this change which can be directed by σ_t.

The GARCH (p, q) model is given by

$$Y_t = \mu + \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_i Y_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-i}^2 \tag{2}$$

where, p and q are orders of GARCH and ARCH respectively which are actually the number of lags and ε_t is the error term which is assumed to be normally distributed with mean zero and conditional variance σ_t^2.

Returns are represented by Y_t and their mean value μ is positive and small. The model parameters are represented by α_0, α_i *and* β_j and they are also relative weights of the lagged terms and usually assumed to be non-negative.

Estimation of parameters with the GARCH approach requires the use of Maximum Likelihood Estimation (MLE) method.

In this paper conditional variance (Volatility) has been estimated using the GARCH (1, 1) Model which is given by

$$Y_t = \mu + \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_j \sigma_{t-1}^2 \tag{3}$$

where Y_{t-1}^2 *and* σ_{t-1}^2 squared residuals and conditional variance of the previous day. The residuals of a return at time t may be given as $Y_t - \mu = \sigma_t \varepsilon_t \Rightarrow R_t = \sigma_t \varepsilon_t$

Volatility of the returns can be derived as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

But $\alpha_0 = \gamma V_L$ where γ is a weight assigned to the long run average variance rate V_L. Since weights must sum to 1,

$$\gamma + \alpha_1 + \beta_1 = 1 \Rightarrow \gamma = 1 - \alpha_1 - \beta_1$$

This implies that

$$V_L = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \tag{4}$$

This means that as the lag increases the variance forecast converges to unconditional variance given by equation (4).

Results and discussion

In order to predict the volatility of a time series data, GARCH model is fitted to the time series data. This is achieved through the estimation of parameters by the Maximum Likelihood Estimation (MLE). During the estimation of unknown parameters, estimation of standard deviation series are calculated recursively using equation 3.

Most financial time series data are associated with fat tailness and volatility clustering which the GARCH models accounts for. Excess Kurtosis can be observed from the probability distributions of assets returns which actually exhibit fatter tails than the Gaussian distribution.

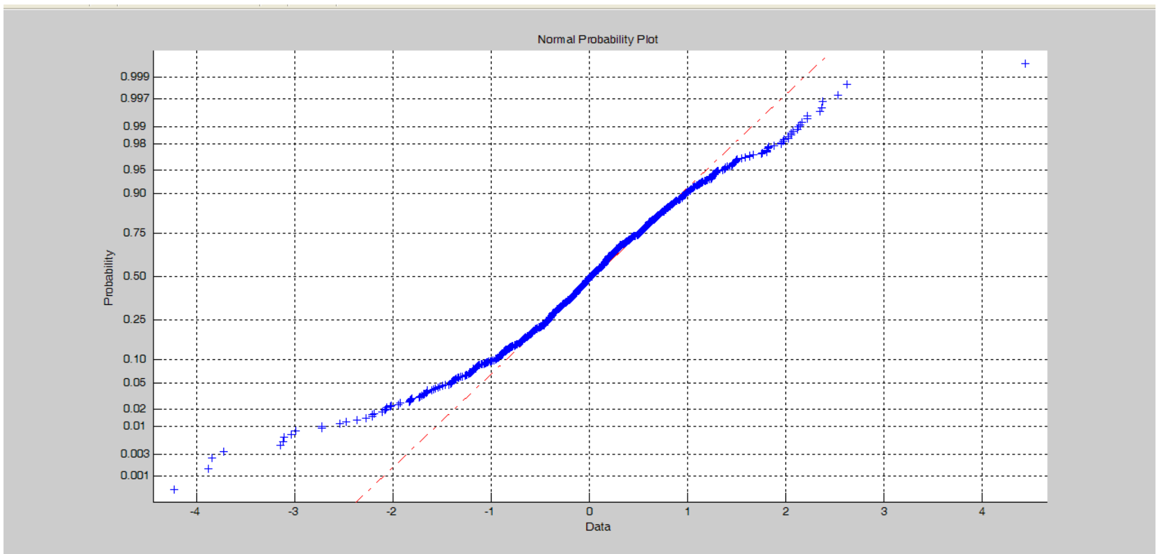

Figure 1: Excess Kurtosis: Blue line illustrates excess kurtosis at the tails of the data and the red line illustrates Gaussian distribution.

Financial Time series also exhibits volatility clustering in which large changes follow large changes and smaller changes follow smaller changes which come as a result of the quality of information reaching the market in clusters (Gallant et al, (1991) and also the time - varying rate of information arrival and the news processing by the market Engle et al, (1990).

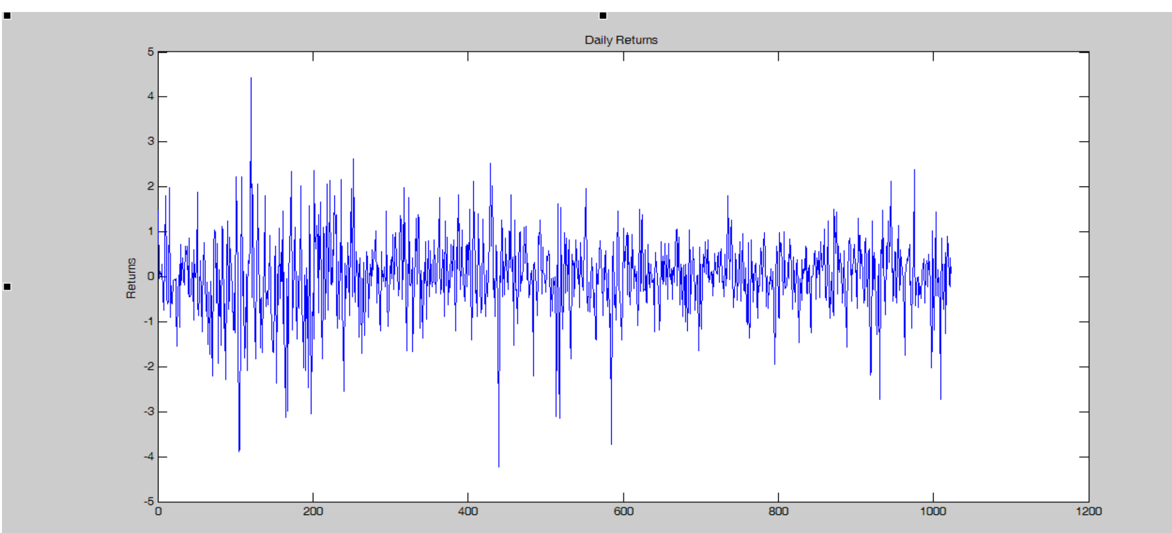

Figure 2: Log Returns of the series showing volatility clustering

Figure 2 shows volatility clustering or persistence which indicates that a serial dependence in the time series data. GARCH models apply the general understanding of volatility dependence to estimate the impacts of previous forecast error and volatility in obtaining the current volatility. This volatility clustering (suggesting changing variance) accounts for excess kurtosis observed in financial data. The correlation of the data can be checked by hypothesis test specifically the Ljung – Box – Pierce Q – test. This implies that when H = 0, it means that no significant correlation exists which means that do not reject the null hypothesis. When H = 1, it means that significant correlation exist which means reject the null hypothesis.

The Q – test statistic is asymptotically Chi – square distributed under the null hypothesis if no serial correlation.

Table 1 below suggest that there is significant correlation presents in the returns on testing up to 10, 15 and 20 lags of the ACF at the 0.05 level of significance.

Table1. Ljung –Box – Pierce Q – test

H	P Value	Test statistic	Critical Value
1.0000	0.2226	44.9440	18.3070
1.0000	0.6160	51.7722	24.9958
1.0000	0.5282	60.8362	31.4104

Estimation parameters and the examination of the estimated GARCH Model

GARCH modeling is significant in this process due to the presence of heteroskedasticity in figure 2.

The estimated parameters and their corresponding standard errors can be observed in table 2 below.

Table2. Estimated Parameters

Parameter	**Value**	**Standard Error**	**T Statistic**
C	0.020418	0.026446	0.7721
K	0.011819	0.0050396	2.3453
GARCH(1)	0.94404	0.012951	72.8932
ARCH(1)	0.040308	0.0079053	5.0989

From Table 2, the conditional mean and the conditional variance (Volatility) model that best fits the observed data is

$$Y_t = 0.020418 + \varepsilon_t \quad \text{and} \quad \sigma_t^2 = 0.011819 + 0.94404\sigma_{t-1}^2 + 0.040308Y_{t-1}^2$$

Since σ_t^2 is a one-period ahead variance forecast based on the past information, it is referred to as conditional variance. From equation (3), α_0 is constant, volatility news from the previous period which is measured as the lag of the squared residuals from the mean equation R_{t-1}^2 and the last periods forecast variance σ_{t-1}^2. It is assumed in GARCH model that effect of a random shock on the most recent volatility declines geometrically over time which is consistent with volatility clustering.

From table 2, it can be observed that most of the information are from the previous days forecast amounting to about 94% and there is a minimal change on the arrival of new information and there is a very small effect on the long run average variance. The long run average variance per day implied by the model is given by equation (4).

That is $V_L = \dfrac{0.011819}{1-0.040308-0.94404} = \dfrac{0.011819}{0.015652} = 0.7551$

Thus the corresponding volatility is given by $\sqrt{0.755111167}$ = 0.86897.

This means that Volatility is 86.897% per day.

Forecasting conditional volatilities for a longer horizons approaches 0.86897. For example if volatilities are forecasted in each period of a 10-period forecast horizon gives the following;

0.8066,0.8076,0.8086,0.8096,0.8105,0.8115,0.8124,0.8133,0.8142,0.8151,0.8160,0.8168,0.8177,0.8185, 0.8193,0.8201,0.8209,0.8217,0.8225,0.8232

This means that as the number of horizons is increased to infinity the conditional volatility approaches the asymptotic value 0.86897 (unconditional volatility).

Figure 3 shows conditional volatilities derived from the fitted returns of the equity prices. This also shows that the assumption of independence and identically distributed is not realistic since financial returns tend to occur in clusters (volatility clustering).

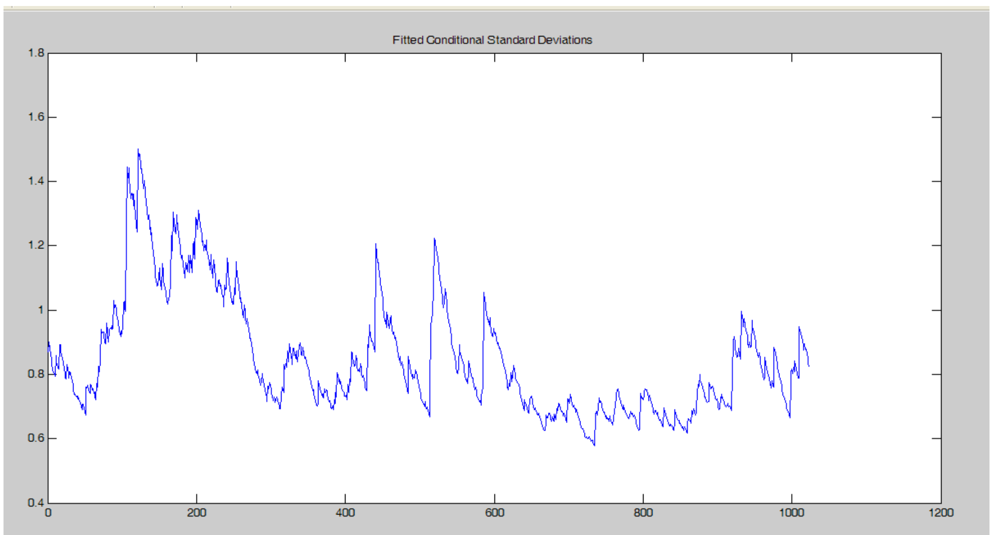

Figure 3: Fitted Conditional Standard Deviations

Conclusion

From the results it can be observed that equities of Barclays Bank of Kenya are highly volatile. This can be observed in the fact that volatility is approximately 87% which confirm the fact that stocks are high risk return on investments. It can also be concluded that strong GARCH effects can be observed in this financial market. Due to volatility clustering assumptions, independent and identically distributed financial returns is not appropriate.

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